

# Thirty-six quantum officers are entangled

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**fwo**



25 May 2026

Joint work with Simeon Ball (UPC)



S. Ball and R. Simoens,  
*Thirty-six quantum officers are entangled*, arXiv:2603.02334.

# Thirty-six officers

Question (Euler, 1782)

*Can six army regiments, each with six officers of different ranks, be arranged in a  $6 \times 6$  square such that no row or column repeats a rank or regiment?*

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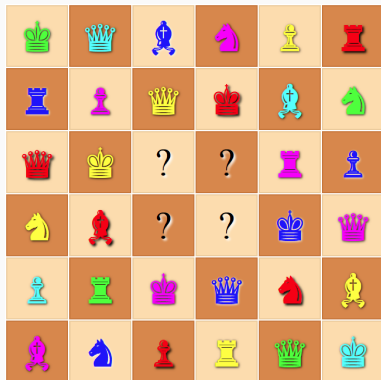


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Theorem (Tarry, 1900)

No.

# Thirty-six entangled officers

Theorem (Rather, Burchardt, Bruzda, Rajchel-Mieldzióć, Lakshminarayan, Życzkowski, 2022)

*There exist entangled orthogonal quantum Latin squares of order six.*

 0 3 0 17	 0 17 17 0	 17 5 5 0	 19 5 0 19	 14 15 18 3	 1 12 3 18
 10 10	 6 0 0 6	 2 7 7 5	 1 14 19 19	 4 9 2 13	 3 18 19 0
 2 13	 2 7	 12 19 1 1	 5 14 15 15	 1 8 4 4	 10 3 16 16
 10 5 4 7	 5 0 17 10	 0 15 14 13	 15 0 7 16	 1 16 10 15	 14 19 5 10
 0 7 0 7	 0 10 13 13	 0 10	 10 14	 2 15 10 10	 5 8 0 8
 10 5 9 16	 15 0 16 13	 14 7 10 10	 3 0 6 6	 0 1	 16 11

# Four faces of the same square

## Entangled orthogonal quantum Latin squares

↔ AME state on four particles

↔ perfect tensor

↔ 2-unitary operator

↔  $((4, 1, 3))_6$  quantum code

MOLS  
MUTUALLY ORTHOGONAL  
LATIN SQUARES

# Latin squares

**LS**( $n$ ): matrix  $A \in \{1, \dots, n\}^{n \times n}$  such that each row and column contains  $n$  different elements.

1	2	3
2	3	1
3	1	2

# Mutually orthogonal Latin squares

$t$  **MOLS**( $n$ ): set  $\{A^1, \dots, A^t\}$  of  $\text{LS}(n)$  such that

$$k_1 \neq k_2 \implies \{(a_{ij}^{k_1}, a_{ij}^{k_2}) : i, j \in \{1, \dots, n\}\} = \{1, \dots, n\}^2.$$

1	2	3
2	3	1
3	1	2

1	2	3
3	1	2
2	3	1

 $\rightarrow$ 

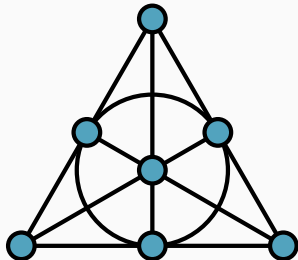
(1, 1)	(2, 2)	(3, 3)
(2, 3)	(3, 1)	(1, 2)
(3, 2)	(1, 3)	(2, 1)

# Mutually orthogonal Latin squares

## Theorem

Let  $n \geq 2$ .

- ▶ There are at most  $n - 1$  **MOLS**( $n$ ).
- ▶  $n - 1$  **MOLS**( $n$ )  $\iff$  projective plane of order  $n$ .



# Quantum Latin squares

**QLS**( $n$ ): matrix  $A \in (\mathbb{C}^n)^{n \times n}$  such that each row and column is an orthonormal basis of  $\mathbb{C}^n$ .

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 = 

$ 1\rangle$	$ 2\rangle$	$ 3\rangle$
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Motivated by the construction of *unitary error bases* [Musto, Vicary, 2016], used in:

- quantum teleportation
- dense coding
- complex Hadamard matrices

# Quantum Latin squares

**QLS**( $n$ ): matrix  $A \in (\mathbb{C}^n)^{n \times n}$  such that each row and column is an orthonormal basis of  $\mathbb{C}^n$ .

$ +\rangle$	$ -\rangle$	$ 3\rangle$
$i -\rangle$	$- 3\rangle$	$ +\rangle$
$i 3\rangle$	$-i +\rangle$	$- -\rangle$

$ 1\rangle$	$ 2\rangle$	$ 3\rangle$	$ 4\rangle$
$ 2\rangle$	$ 1\rangle$	$ 4\rangle$	$ 3\rangle$
$ 3\rangle$	$ 4\rangle$	$ +\rangle$	$ -\rangle$
$ 4\rangle$	$ 3\rangle$	$ -\rangle$	$ +\rangle$

where  $|+\rangle = \frac{|1\rangle+|2\rangle}{\sqrt{2}}$  and  $|-\rangle = \frac{|1\rangle-|2\rangle}{\sqrt{2}}$ .

# Quantum Latin squares

$$\mathbf{LS}(n) \subsetneq \mathbf{QLS}(n)$$

$$\{1, \dots, n\} \quad \mathbb{C}^n$$

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$$t \mathbf{MOLS}(n) \subsetneq t \mathbf{MOQLS}(n) \subsetneq t \mathbf{MEQLS}(n)$$

$$\{1, \dots, n\}^t \quad (\mathbb{C}^n)^t \quad (\mathbb{C}^n)^{\otimes t}$$

MOQLS  
MUTUALLY ORTHOGONAL  
QUANTUM LATIN SQUARES

# Mutually orthogonal quantum Latin squares

$t$  **MOQLS**( $n$ ): set  $\{A^1, \dots, A^t\}$  of **QLS**( $n$ ) such that

$$k_1 \neq k_2 \implies \left\{ a_{ij}^{k_1} \otimes a_{ij}^{k_2} : i, j \in \{1, \dots, n\} \right\} \text{ is a basis of } \mathbb{C}^n \otimes \mathbb{C}^n.$$

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$ 1\rangle$	$ 2\rangle$	$ 3\rangle$	$\rightarrow$	$ 1\rangle \otimes  1\rangle$	$ 2\rangle \otimes  2\rangle$	$ 3\rangle \otimes  3\rangle$
$ 2\rangle$	$ 3\rangle$	$ 1\rangle$		$ 2\rangle \otimes  2\rangle$	$ 3\rangle \otimes  1\rangle$	$ 1\rangle \otimes  2\rangle$
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$ 3\rangle$	$ 1\rangle$	$ 2\rangle$		$ 2\rangle$	$ 3\rangle$	$ 1\rangle$		$ 3\rangle \otimes  2\rangle$	$ 1\rangle \otimes  3\rangle$	$ 2\rangle \otimes  1\rangle$

► Motivated by the construction of *mutually unbiased bases*

# Mutually orthogonal quantum Latin squares

Theorem (Musto, Vicary, 2019)

Let  $n \geq 2$ . There are at most  $n - 1$  **MOQLS**( $n$ ).

Theorem (Ball, Simoens, 2026+)

$n - 1$  **MOQLS**( $n$ ) are classical.

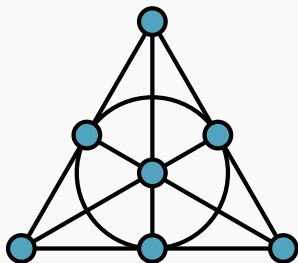
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MEQLS

MUTUALLY ENTANGLED QUANTUM  
LATIN SQUARES

# Mutually entangled quantum Latin squares

$t$  **MEQLS**( $n$ ): matrix  $A \in \left( \underbrace{\mathbb{C}^n \otimes \cdots \otimes \mathbb{C}^n}_t \right)^{n \times n}$  such that

$$\frac{1}{n} \sum_{ij} |i\rangle \otimes |j\rangle \otimes a_{ij}$$

is *maximally mixed* over all subsystems of size two.

# Quantum Latin squares

2 MOLs(3) :

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2 MOQLs(3) :

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2 MEQLs(3) :

$ 11\rangle$	$ 22\rangle$	$ 33\rangle$
$ 23\rangle$	$ 31\rangle$	$ 12\rangle$
$ 32\rangle$	$ 13\rangle$	$ 21\rangle$

# Entanglement is powerful

## Theorem

$t$  **MOLS**( $n$ )  $\iff (t + 2, n^2, t + 1)_n$  code.

## Theorem (Ball, Simoens, 2026+)

$t$  **MEQLS**( $n$ )  $\iff ((t + 2, 1, \geq 3))_n$  quantum code.

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$\frac{1}{\sqrt{2}} \left(  0000\rangle +  1110\rangle \right)$	$\frac{1}{\sqrt{2}} \left(  0101\rangle +  1011\rangle \right)$
$\frac{1}{\sqrt{2}} \left(  0011\rangle +  1101\rangle \right)$	$\frac{1}{\sqrt{2}} \left(  0110\rangle +  1000\rangle \right)$

4 **MEQLS**(2)

# Quantum Latin squares

$\nexists$  2 **MOLS(6)**

[Tarry, 1900]

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[Rather et al., 2022]

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2 **MOQLS**(6): pair  $\{A, B\}$  of **QLS**( $n$ ) such that

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THIRTY-SIX OFFICERS  
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- *Latin square graph* of **LS(6)** has no *orthonormal representation* in  $\mathbb{C}^6$

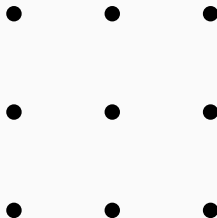


# Latin square graphs

The **Latin square graph** of a  $LS(n)$

- ▶ Vertices:  $n^2$  entry coordinates of the LS.
- ▶ Edges: same row or column or equal entries.

1	2	3
2	3	1
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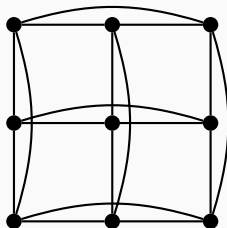


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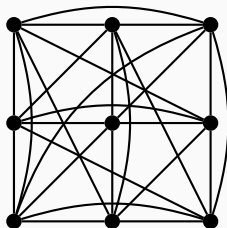


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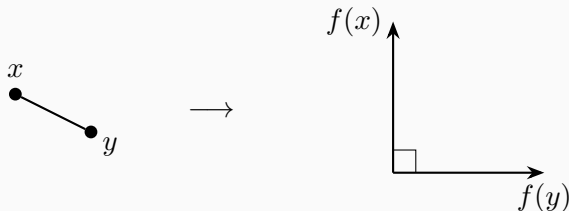
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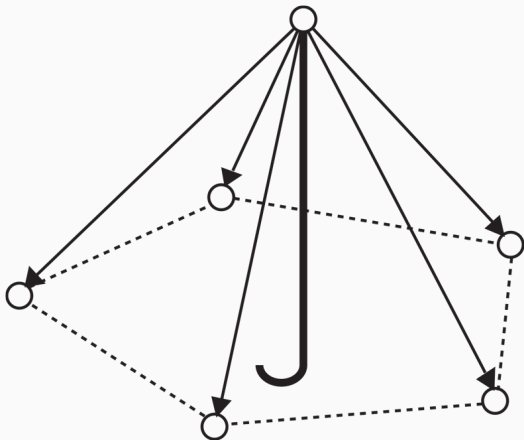


# Orthonormal representations

An **orthonormal representation** of a graph  $G$  in  $\mathbb{C}^n$  is a map  $f : V(G) \rightarrow \mathbb{C}^n$  such that  $x \sim y \implies f(x)$  and  $f(y)$  are orthonormal.



# Orthonormal representations



Lemma (Ball, Simoens, 2026+)

*If a  $\mathbf{LS}(n)$  can be extended to  $2 \mathbf{MOQLS}(n)$ , its Latin square graph has an orthonormal representation in  $\mathbb{C}^n$ .*

# Orthonormal representations

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If a  $\mathbf{LS}(n)$  can be extended to  $\mathbf{2\ MOQLS}(n)$ , its Latin square graph has an orthonormal representation in  $\mathbb{C}^n$ .

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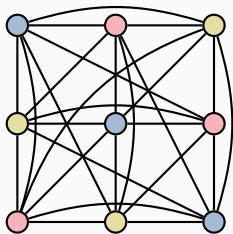
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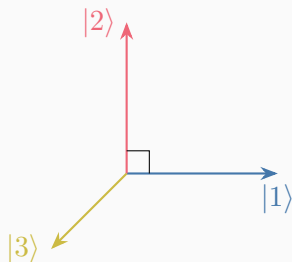
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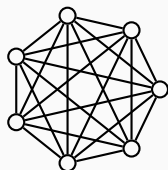


$f \rightarrow$



# Orthonormal representations

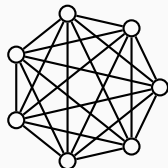
Recipe to disprove the existence of an orthonormal representation in  $\mathbb{C}^6$ :



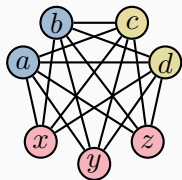
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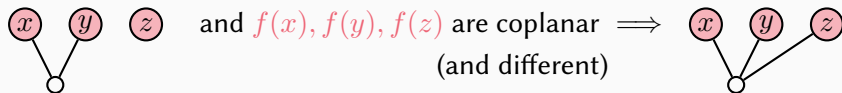
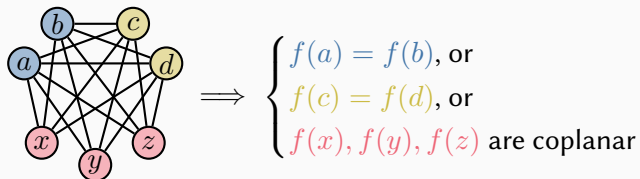
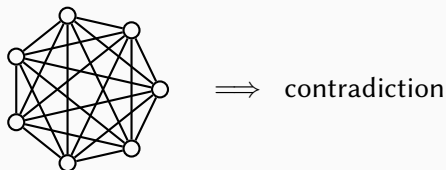
$\implies$  contradiction



$\implies \begin{cases} f(a) = f(b), \text{ or} \\ f(c) = f(d), \text{ or} \\ f(x), f(y), f(z) \text{ are coplanar} \end{cases}$

# Orthonormal representations

Recipe to disprove the existence of an orthonormal representation in  $\mathbb{C}^6$ :



# Orthonormal representations

Theorem (Ball, Simoens, 2026+)

2 **MOQLS(6)** do not exist.

*Proof.*

- One of 2 **MOQLS(6)** is **classical**.
- Latin square graph of **LS(6)** has no orthonormal representation in  $\mathbb{C}^6$ .

1	2	3	4	5	6
2	1	4	3	6	5
3	4	5	6	1	2
4	3	6	5	2	1
5	6	1	2	3	4
6	5	2	1	4	3

1	2	3	4	5	6
2	1	4	3	6	5
3	4	5	6	1	2
4	3	6	5	2	1
5	6	1	2	4	3
6	5	2	1	3	4

1	2	3	4	5	6
2	1	4	3	6	5
3	4	5	6	1	2
4	5	6	1	2	3
5	6	1	2	3	4
6	3	2	5	4	1

1	2	3	4	5	6
2	1	4	3	6	5
3	4	5	6	1	2
4	5	6	2	3	1
5	6	2	1	4	3
6	3	1	5	2	4

1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	2	4
4	6	2	5	1	3
5	3	6	1	4	2
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1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	2	4
4	6	2	5	1	3
5	3	6	2	4	1
6	4	5	1	3	2

1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	2	4
4	6	2	5	3	1
5	4	6	2	1	3
6	3	5	1	4	2

1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	2	4
4	6	5	1	3	2
5	4	6	2	1	3
6	3	2	5	4	1

1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	4	2
4	6	5	1	2	3
5	3	6	2	1	4
6	4	2	5	3	1

1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	4	2
4	6	5	1	2	3
5	4	6	2	3	1
6	3	2	5	1	4

1	2	3	4	5	6
2	1	4	5	6	3
3	4	2	6	1	5
4	6	5	2	3	1
5	3	6	1	2	4
6	5	1	3	4	2

1	2	3	4	5	6
2	3	1	5	6	4
3	1	2	6	4	5
4	6	5	2	1	3
5	4	6	3	2	1
6	5	4	1	3	2

# Orthonormal representations

Theorem (Ball, Simoens, 2026+)

2 **MOQLS(6)** do not exist.

*Proof.*

- One of 2 **MOQLS(6)** is **classical**.
- Latin square graph of **LS(6)** has no orthonormal representation in  $\mathbb{C}^6$ .

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1	2	3	4	5	6
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3	5	1	6	2	4
4	6	5	1	3	2
5	4	6	2	1	3
6	3	2	5	4	1

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2	1	4	3	6	5
3	5	1	6	4	2
4	6	5	1	2	3
5	3	6	2	1	4
6	4	2	5	3	1

1	2	3	4	5	6
2	1	4	3	6	5
3	5	1	6	4	2
4	6	5	1	2	3
5	4	6	2	3	1
6	3	2	5	1	4

1	2	3	4	5	6
2	1	4	5	6	3
3	4	2	6	1	5
4	6	5	2	3	1
5	3	6	1	2	4
6	5	1	3	4	2

1	2	3	4	5	6
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3	1	2	6	4	5
4	6	5	2	1	3
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6	5	4	1	3	2

Theorem (Ball, Simoens, 2026+)

**2 MOQLS(6)** *do not exist.*

# Conclusion

Theorem (Ball, Simoens, 2026+)

*2 MOQLS(6) do not exist.*

Theorem (Han, Zang, Zhang, Tian, 2025)

*If  $n \geq 4$ , there exist non-classical 2 MOQLS( $n$ ), except possibly for  $n \in \{4, 5, 6, 7\}$ .*

# Conclusion

Theorem (Ball, Simoens, 2026+)

2 **MOQLS(6)** *do not exist*.

Theorem (Han, Zang, Zhang, Tian, 2025, Ball, Simoens, 2026+)

If  $n \geq 4$ , there exist non-classical 2 **MOQLS( $n$ )**, except possibly for  $n \in \{4, 5, 6, 7\}$   $n = 7$ .

# Conclusion

Theorem (Ball, Simoens, 2026+)

$2 \text{ MOQLS}(6)$  do not exist.

Theorem (Han, Zang, Zhang, Tian, 2025, Ball, Simoens, 2026+)

If  $n \geq 4$ , there exist non-classical  $2 \text{ MOQLS}(n)$ , except possibly for  $n \in \{4, 5, 6, 7\}$   $n = 7$ .

Open problems:

- Are  $2 \text{ MOQLS}(7)$  classical?

# Conclusion

Theorem (Ball, Simoens, 2026+)

$2$  **MOQLS**(6) *do not exist*.

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Open problems:

- Are  $2$  **MOQLS**(7) classical?
- Find a “least entangled” solution of thirty-six officers.

Thank you for listening!



S. Ball and R. Simoens,  
*Thirty-six quantum officers are entangled*, arXiv:2603.02334.